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Establishing an evaluation performance measurement model for the financial service sector

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The study employs a multiple criteria decision making (MCDM) approach to evaluate the performance of wealth management (WM) banking. We first apply the balanced scorecard methodology to build a framework for WM banking and then provide a framework for the sub-criteria by applying the Delphi method. The empirical results reveal that the performance of WM banks in Taiwan is evaluated by applying the analytic network process (ANP) and grey relational analysis (GRA). Importantly, the proposed model can assist the banking sector in assessing the performance of WM banks, making it highly applicable for bank managers engaged in decision-making as well as financial sector analysts.

Key words: Wealth management, balanced scorecard, analytic network process (ANP), grey relational analysis (GRA).

INTRODUCTION

Taiwan's financial banking markets have changed dramatically, particularly following the Asian Financial Crisis in 1997. The banking business sectors in Taiwan involve four main areas, including consumer, corporate, wealth management (WM) and investment banking. The WM banking sector is actively promoted in order to reap a risk-free premium. In addition, the organizational structure of the WM service sector has improved as banks have implemented management practices that have helped to increase their competitiveness. In such a competition-intensive environment, evaluating the performance of WM banking has become a priority. During the process of evaluating a WM bank, apart from legal issues, policy-makers and business groups seldom consider potential factors that could impact a bank's competitiveness.

For their part, bank managers are realizing that the business performance (Hernando and Nieto, 2007) records of their institutions can be used to gain a competitive advantage. One of the key aspects of this paper is its evaluation of the various alternatives available to Taiwan's banking sector in the area of WM. One such option, involving the application of a systemic analysis technique, is presented in this paper. This approach evaluates various dimensions of business performance through an analytical hierarchy network model. The analytic hierarchy process (AHP) is one of the tools that can be used to handle a multi-criteria decision-making (MCDM) problem (Saaty, 1990; Javalgi et al., 1989; Wu et al., 2010). However, a shortcoming of AHP is that it is lacking in the consideration of interdependencies, if any, among the selection criteria. The ANP is a similar technique, but it is able to capture the interdependencies among the criteria under consideration, allowing for a more systemic analysis.

It allows for the inclusion of criteria, both tangible and intangible (more difficult to quantify), which have some bearing on making the best decision (Saaty, 1996; Wu et al., 2009). In addition, many of the criteria have some level of interdependency, thus making ANP modeling a more effective methodology.

The proposed model presented in this paper structures the problem related to evaluating an alternative to the performance for WM banks in a hierarchical form and links the determinants, dimensions, and enablers of performance with the various alternatives. One of the important issues for any strategic planner would be how the organization should prioritize the determinants and what policy elements or initiatives impact them. Javalgi et al. (1989) applied AHP to evaluate and select consumer bank decisions, while Arbel and Orger (1990) also used

The balanced scorecard (BSC) is a performance measurement system (Mukherjee and Varela, 1993; Lipe and Salterio, 2000; Chenhall, 2005; Kim and Davidson, 2004; Gumbus and Lussier, 2006; Henri, 2006 Wong-On-Wing et al., 2007) that allows managers to look at a business from four perspectives: financial aspects, customer viewpoints, internal business processes, and learning and growth objectives (Kaplan and Norton, 1992; Lipe and Salterio, 2002; Ravi et al., 2005) combined the BSC and an ANP-based approach to provide a more accurate representation of the issue of conducting reverse logistics operations for end-of-life computers. Searcy (2004) aligned the BSC with a firm's strategic approach using the AHP. Chan (2006) applied the AHP and BSC approach to evaluate the organizational performance of healthcare facilities. In the proposed model, the dimensions of the performance for the WM banks are based on the four perspectives of the BSC, to balance as well as link the financial and non-financial, tangible and intangible, and internal and external factors. Therefore, the proposed framework provides an integrated approach to the evaluated MCDM problem for WM banks.

This paper proposes an approach to evaluating the performance of WM banks that combines the ANP and GRA. The ANP is used in obtaining the relative weights of the criteria, and the GRA approach is used to rank the WM banks' performance in terms of their overall performance based on multiple evaluation criteria. The GRA and ANP-based decision-making methods for constructing an evaluation method can provide bank decision makers or the financial sector with a valuable reference for evaluating performance.

THE BALANCED SCORECARD

Kaplan and Norton (1992, 1993, 1996a) first presented the balanced scorecard (BSC) concept in a series of articles in the Harvard Business Review. They argued that traditional financial accounting measures such as return-on-investment and payback period analysis (Johnson and Kaplan, 1987; Kaplan and Norton, 1996a; Nørreklit, 2003; Frigo and Krumwiede, 2000) offer a narrow and incomplete picture of business performance, and that reliance on such data hinders the creation of future business value. Accordingly, they suggested that financial measures can be supplemented with additional ones that reflect customer satisfaction, internal business processes, and the ability to learn and grow. The balanced scorecard was designed to complement “financial measures of past performance with measures of the drivers of future performance” (Kaplan and Norton, 1996a; Ittner et al 2003; Lin et al., 2005).

From its name, the BSC concept reflects the intent to keep score on a set of items that maintain a balance between “short- and long-term objectives, between financial and non-financial measures (Ittner and Larcker, 1998; Cebeci, 2008; Garcia-Valderrama et al., 2008; Chang et al., 2008), between lagging and leading indicators, and between internal and external performance perspectives” (Kaplan and Norton, 1996a). A focus of management on such a broad set of performance measures should not only help to ensure good short-term financial results, but also guide a business as it seeks to achieve its strategic goals (Martinsons et al., 1999; Brewer and Speh, 2000; McWhorter, 2003).

The exclusive focus on standard accounting measures such as the return-on-investment and payback period analysis within the financial industry has been criticized as the root cause of many problems. With their emphasis on short-term financial performance metrics, managers tend to sacrifice the pursuit of endeavors that can result in long-term benefits, including new product development, process improvement, human resource development, information technology, and customer and market development, in the interest of current profitability, thus limiting investment in future growth opportunities (Sudarsanam et al., 1996; Banker et al., 2004; Ellat et al., 2008). Such actions by managers are a consequence of poorly designed performance measurement systems focusing solely on short-term financial performance. By attempting to resolve the problem by supplementing standard financial practices with additional indicators that can help a firm evaluate its long-term performance, Kaplan and Norton introduced the BSC to integrate the e-business performance assessment of a company (Yasuo, 1992; Murphy et al., 1996; Loughran and Ritter, 1997; Morgan and Strong, 2003; Lee et al., 2008; Huang, 2009).

Of the four key performance perspectives of the BSC, only one involves standard financial performance indicators, whereas the other three contain non-financial performance measurement perspectives: customer satisfaction, internal business processes, and learning and growth capacities. These four perspectives constitute the framework of the BSC (Figure 1). Although the BSC is intended primarily as a measure of system performance, this paper examines the model as a methodology by which the WM development banking sector creates an integrated framework for attracting high net worth customers. The four dimensions of this model and their associate enablers are briefly discussed below (Kaplan and Norton, 1996b).

Financial perspective

The BSC retains the financial perspective, as financial data are valuable in calculating the readily measurable
As customers are the source of business profits, satisfying customer needs is the ultimate objective of enterprises. With this perspective, management determines the expected target customers and market segments for operational units, and monitors the performance of operational units in these target segments. Examples of core performance measures are customer satisfaction, customer retention, new customer acquisition, market position, and market share in targeted segments.

In devising the customer perspective, the banks consider the following possible measurements: customer acquisition, VIP-certified financial status, customer profitability, customer confidence, and customer retention.

**Internal business process perspective**

The objective of this perspective is to satisfy shareholders and customers by excelling in some business processes that have the greatest impact. In determining the objectives and measures, the first step should be to incorporate value-chain analysis. An outmoded operating process should be adjusted to factor-in financial and customer dimension objectives. A complete internal business-process value chain that can meet the current and future needs should then be constructed. The internal value chain of a common enterprise consists of three main business processes: innovation, operation, and after-sales services.

The banks evaluate performance on the basis of: innovation in system programming, certification of a financially integrated professional platform, the operating quality of a group of customers, internal customer satisfaction, and management stratum support.

**Learning and growth perspective**

The BSC’s learning and growth element is intended to identify the criteria for establishing the infrastructure of an economic consequences of previous actions. Measures of financial performance indicate whether a company’s strategy, implementation, and execution contribute to bottom-line improvement. Financial objectives typically relate to measures of profitability, including operating income, return on capital, and economic value added. Alternative financial objectives can be rapid sales growth or the generation of cash flow.

Financial indicators that the banks regularly review include: the handling of charge revenue, customers’ market share ratios, the ability to achieve profitability, and asset management.

**Customer perspective**

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In devising the customer perspective, the banks consider the following possible measurements: customer acquisition, VIP-certified financial status, customer profitability, customer confidence, and customer retention.
Step 1: Establish an evaluation framework

Based on Delphi Method and Balanced Scorecard

Step 2: Determine the weight of criteria and sub-criteria by using ANP

Based on ANP

Step 3: Determine the weight of wealth management in bank performance by using GRA

Based on GRA

Step 4: Final performance score

Based on GRA

Figure 2. ANP and GRA measurement processes for wealth management in bank performance.

organization’s growth. This indicator is arguably the most critical of the balanced scorecard perspectives for addressing the future needs of an organization. It may also be the most difficult parameter to measure. As Kaplan and Norton (1996a) pointed out: “managers in several organizations have noted that when they were evaluated solely on short-term financial performance, they often found it difficult to sustain investments to enhance the capability of their people, systems, and organizational processes.”

In devising the learning and growth perspective, banks consider the following possible measurements: the wealth managers’ professional knowledge, the education and training of wealth managers, the size of the wealth managers’ team, the wealth managers’ complaint system, and the appropriateness of performance policies for rewards and punishments.

EVALUATION MODEL

The evaluation procedure adopted in this study consists of several steps as shown in Figure 2. The first step is to identify the multiple criteria that are considered in the decision making process for the decision makers to make an objective and unbiased decision. The Delphi method is adopted here not only to accumulate expert opinions, but also to identify the determinants of the integrated marketing communication-based model. After constructing a criteria framework, the criteria weights can be calculated by using ANP. Finally, we adopt the GRA approach to achieve the final ranking results. The detailed descriptions of each step are elaborated further.

Establishing an evaluation model and defining the evaluative criteria

Based on the Delphi method, a general consensus among experts can be reached to establish a model. The ultimate goal of evaluating the ideal performance can be achieved, followed by four-evaluation criterion, nineteen sub-criteria and finally, the alternatives and the criteria and sub-criteria.

Financial perspective (C_1)

The financial perspective refers to a WM bank’s revenue during the operation time, including handling charges/revenue, the customer market share ratio, the capacity for profitability, and the management of assets (as suggested by the bank managers).

i) Handling charges/revenue (SC_1): The WM banks’ revenue earned by selling the customer’s products.

ii) Customers market share ratio (SC_2): This reflects the proportion of business in a given market (in terms of the number of customers, dollars spent, or unit volume sold) that a business unit sells.

iii) Capacity for profitability (SC_3): The various products and projects created by WM banks that serve to increase their capacity for profitability.

iv) Asset management (SC_4): Asset management may be
defined as a comprehensive and structured approach to the long-term management of assets as tools for the efficient and effective delivery of community benefits.

Customer perspective (Cᵢ)

Factors involving the customers of WM banks include customer acquisition, VIP-certified financial services (as suggested by bank managers), customer profitability, customer confidence, and customer retention (Kaplan and Norton, 1996a):

i) Customer acquisition (SC₃): Measures, in absolute or relative terms, of the rate at which a business unit attracts or wins new customers or business.

ii) VIP-certified financial services (SC₄): The provision to customers of complete VIP-certified financial services.

iii) Customer profitability (SC₅): Measures the net profit of a customer, or of a segment, after allowing for the unique expenses required to support that customer or segment.

iv) Customer confidence (SC₆): Wealth managers improve the VIP service for new customers and assure them of confidentiality.

v) Customer retention (SC₇): This tracks, in absolute or relative terms, the rate at which a business unit retains or maintains ongoing relationships with its customers.

Internal business process perspective (Cᵢ)

The WM banks’ performance measurement, organization, management practices, and competitors all influence the following factors: the lead-in innovation system programming, a certified financial integration platform for professionals, the operating quality for a group of customers, internal customer satisfaction and management stratum support (as suggested by bank managers):

i) Lead-in innovation system programming (SC₁₀): Measures how well a WM bank accedes to innovation system programming in accordance with the wealth manager’s professional knowledge and ability.

ii) Certified financial integration platform for professionals (SC₁₁): The WM bank provides the certified financial integration platform for professional VIP customers.

iii) Operating quality for a group of customers (SC₁₂): The administration of the operational quality for a group of customers requires a professional CEO and wealth managers.

iv) Internal customer satisfaction (SC₁₃): The internal customer satisfaction index was constructed through monthly surveys of randomly selected customers in the WM bank’s targeted segments.

v) Management stratum support (SC₁₄): To obtain the executive manager’s support.

Learning and growth perspective (Cᵢ)

The factors comprising the learning and growth perspective include the wealth manager’s professional knowledge and growth, the education and training of the WM team, the size of the wealth manager’s team, the wealth manager’s complaint system, and the appropriateness of the performance policy for rewards and punishments:

i) Wealth managers’ professional knowledge and growth (SC₁₅): The WM bank develops the wealth manager’s professional knowledge and techniques.

ii) Education and training of the wealth management team (SC₁₆): This includes wealth management knowledge, the use of teleconferencing, and the attainment of basic professional certificates.

iii) Size of the wealth manager’s team (SC₁₇): The bank provides a complete wealth management team for the customer service system.

iv) Wealth managers’ complaint system (SC₁₈): The banks provide a complaint system for the wealth managers.

v) Appropriateness of performance policy rewards and punishments (SC₁₉): Herzberg’s motivation theory (1966) argued that there must be linkages among striving, performance, and remuneration. Academicians have also indicated that linking bounty, remuneration and performance to the performance bonus system can enable wealth managers to work more effectively with increased motivation.

Determining the weight of criteria using ANP

Whereas AHP represents a framework with a unidirectional hierarchical relationship (Wu et al., 2009), ANP allows for more complex interrelationships among decision levels and attributes. The ANP feedback approach replaces hierarchies with networks, in which the relationships between levels are not easily represented as higher or lower, dominated, or being dominated, directly or indirectly (Meade and Sarkis, 1999; Saaty, 1999, 2006; Wu et al., 2009). For instance, not only does the importance of the criteria determine the importance of the alternatives as in a hierarchy, but also the importance of the alternatives may have an impact on the importance of the criteria (Saaty, 1996, 2006; Wu et al., 2010).

In ANP, as in AHP, decision elements for each component are compared pair-wise with respect to their importance to their control criteria, while the components themselves are also compared pair-wise with respect to their contribution to the goal. Decision makers are asked to respond to a series of pair-wise comparisons where two elements or two components at a time will be compared in terms of how they contribute to their particular upper level criterion (Meade and Sarkis, 1999; Saaty, 2006). In addition, if there are interdependencies among elements of a component, pair-wise comparisons also need to be...
created, and an eigenvector can be obtained for each element to show the influence of other elements on it. The relative importance values are determined on a scale of 1 to 9. A reciprocal value is assigned to the inverse comparison; that is, \( a_{ij} = 1/\ a_{ji} \) where \( a_{ij} \) (a component) denotes the importance of the \( j^{th} \) (jth) element compared to the \( i^{th} \) (ith) element. The pair-wise comparison in ANP is featured in the framework of a matrix, and a local priority vector can be derived as an estimate of relative importance associated with the elements (or components) being compared by solving the following formula:

\[
A \cdot w = \lambda_{\text{max}} \cdot w
\]

where \( A \) is the pair-wise comparison matrix, \( w \) is the eigenvector, and \( \lambda_{\text{max}} \) is the largest eigenvalue of \( A^2 \).

Furthermore, to obtain global priorities in a system with interdependent influences, the local priority vectors are entered in the appropriate columns of a matrix, known as a super-matrix (Appendix A).

The super-matrix concept is similar to the Markov chain process (Saaty, 1996). As a result, a super-matrix is actually a partitioned matrix, where each matrix segment represents a relationship between two nodes (components or clusters) in a system (Meade and Sarkis, 1999).

However, each column of the super-matrix itself may include several sub-columns each with its own priority element, which must be normalized and synthesized to account for the overall components’ influence by column. This process makes the super-matrix column stochastic. The final priority weights - which account for element interactions - are derived by multiplying the super-matrix by itself until the columns stabilize, which occurs when the super-matrix entries become identical across each row or cycle in blocks in which case one uses what is known as Cesaro summability, and the result is known as the limiting matrix. The final priority weights are extracted from this limiting matrix. In essence, this solution algorithm derives weights that account for interactions among components, which is a clear benefit of the dynamic ANP model over static models.

Determining the WM bank’s performance by using GRA

The concept of grey relational space is based on the combined concepts of system theory, space theory and control theory (Deng, 1982). It can be used to capture the correlations between the references factor and other compared factors of a system (Deng, 1988). One of the features of GRA is that both qualitative and quantitative relationships can be identified among complex factors with insufficient information (relative to conventional statistical methods). Under such a condition, the results generated by conventional statistical techniques may not be acceptable without sufficient data to achieve desired confidence levels. By contrast, grey system theory can indeed identify major correlations among factors of a system with a relatively small amount of data.

The procedure for calculating the GRA is as follows. Let \( X_0 \) be the referential series with \( K \) entities (or criteria) of \( X_1, X_2, ..., X_K \) or \( N \) measurement criteria. Then:

\[
X_0 = \{x_0(1), x_0(2), ..., x_0(j), ..., x_0(K)\}
\]

\[
X_i = \{x_i(1), x_i(2), ..., x_i(j), ..., x_i(K)\}, \quad i = 1, 2, ..., M
\]

where \( M \) is the total number of series.

The grey relational coefficient between the compared series \( X_i \) and the referential series of \( X_0 \) for the \( j^{th} \) entity is defined as:

\[
\gamma_{0j} = \frac{\Delta_{\min} + \Delta_{\max}}{\Delta_{0j}(i) + \Delta_{\max}}, \quad (1)
\]

where \( \Delta_{0j}(j) \) is the absolute value of the difference between \( X_0 \) and \( X_i \) for the \( j^{th} \) entity, that is, \( \Delta_{0j}(j) = |x_0(j) - x_i(j)| \), and \( \Delta_{\max} = \max_j \Delta_{0j}(j) \), \( \Delta_{\min} = \min_j \Delta_{0j}(j) \).

The grey relational grade (GRG) for the series of \( X_i \) is given as:

\[
\Gamma_{0i} = \sum_{j=1}^{K} w_j \gamma_{0j}(j), \quad (2)
\]

where \( w_j \) is the weight attached to the \( j^{th} \) entity. If it is not necessary to apply the weight, \( w_j = 1/K \) is used for averaging.

Before calculating the grey relational coefficients, the data series can be treated based on the three kinds of

\[\text{C.R.} \leq 0.1 \]

A score of 1 represents equal importance between the two elements and a score of 9 indicates the extreme importance of one element (row component in the matrix) compared to the other one (column component in the matrix).

1 A score of 1 represents equal importance between the two elements and a score of 9 indicates the extreme importance of one element (row component in the matrix) compared to the other one (column component in the matrix) (Meade and Sarkis, 1999; Saaty, 2006).

2 If \( A \) is a consistency matrix, eigenvector \( X \) can be calculated by \( (A - \lambda_{\text{max}} I)X = 0 \). Saaty (1980) proposed utilizing a consistency index (C.I) and a consistency ratio (C.R) to verify the consistency of the comparison matrix. C.I. and R.I. are defined as follows: \( C.I. = \frac{\lambda_{\text{max}} - n}{n - 1} \), where \( R.I. \) represents the average consistency index over numerous random entries of same-order reciprocal matrices. If \( C.R. \leq 0.1 \), the estimate is accepted; otherwise, a new comparison matrix is solicited until \( C.R. \leq 0.1 \).
situation (Appendix B) and the linearity of data normalization to avoid distorting the normalized data (Hsia and Wu, 1997).

CASE IMPLEMENTATION

Step 1: Establishing an evaluation framework

Using the Delphi method, the first step is to acquire inner interdependence. The dependencies among the criteria and sub-criteria are depicted in Figures 3 and 4.

Step 2: Determining the weight of criteria using the ANP

The weights for the criteria and sub-criteria are then determined for a sample group of 19 individuals that match the characteristics earlier mentioned with each respondent making a pair-wise comparison of the decision criteria and sub-criteria in order to assign them relative scores. The relative scores provided by the eleven experts are aggregated using the geometric mean method. Table 1 describes the aggregate pair-wise comparison matrix for the $W_{21}$ (criteria). The priorities for
Table 1. Performance aggregate pair-wise comparison matrix for $W_{21}$.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_4$</th>
<th>Eigenvectors (weights)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$</td>
<td>1.000</td>
<td>1.340</td>
<td>2.240</td>
<td>3.450</td>
<td>0.392</td>
</tr>
<tr>
<td>$C_2$</td>
<td>0.746</td>
<td>1.000</td>
<td>2.580</td>
<td>1.520</td>
<td>0.301</td>
</tr>
<tr>
<td>$C_3$</td>
<td>0.446</td>
<td>0.388</td>
<td>1.000</td>
<td>2.350</td>
<td>0.184</td>
</tr>
<tr>
<td>$C_4$</td>
<td>0.290</td>
<td>0.658</td>
<td>0.426</td>
<td>1.000</td>
<td>0.123</td>
</tr>
</tbody>
</table>

The performance criteria, $W_{21}$, can be obtained by the procedure stated earlier and is:

$$W_{21} = \begin{bmatrix} C_1 & 0.392 \\ C_2 & 0.301 \\ C_3 & 0.184 \\ C_4 & 0.123 \end{bmatrix}$$ (3)

Equation 3 reveals that the respective weights of the four evaluative criteria are $C_1$ (0.392), $C_2$ (0.301), $C_3$ (0.184), and $C_4$ (0.123).

The eigenvectors for the financial perspective, customer perspective, internal business process perspective and learning and growth perspective are organized into a matrix, $W_{32}$ (sub-criteria), which represent the relative importance of the sub-criteria with respect to their upper level criteria:

The respective weights of the four sub-criteria for the financial perspective are $SC_1$ (0.251), $SC_2$ (0.164), $SC_3$ (0.421), and $SC_4$ (0.164).

The respective weights of the five sub-criteria for the customer perspective are $SC_5$ (0.112), $SC_6$ (0.255), $SC_7$ (0.189), $SC_8$ (0.255), and $SC_9$ (0.189).

The respective weights of the five sub-criteria for the internal business process perspective are $SC_{10}$ (0.146), $SC_{11}$ (0.255), $SC_{12}$ (0.197), $SC_{13}$ (0.147), and $SC_{14}$ (0.255).

The respective weights of the five sub-criteria for the learning and growth perspective are $SC_{15}$ (0.320), $SC_{16}$ (0.200), $SC_{17}$ (0.095), $SC_{18}$ (0.243), and $SC_{19}$ (0.142).

The Delphi method is used to examine the inner interdependence, that is, the dependencies among the criteria and sub-criteria.

The resulting eigenvectors obtained from the pair-wise comparisons formed matrix, $W_{22}$, are:

$$W_{22} = \begin{bmatrix} C_1 & 0.251 & 0 & 0 & 0 \\ C_2 & 0.164 & 0 & 0 & 0 \\ C_3 & 0.421 & 0 & 0 & 0 \\ C_4 & 0.164 & 0 & 0 & 0 \\ SC_{10} & 0 & 0.112 & 0 & 0 \\ SC_6 & 0 & 0.255 & 0 & 0 \\ SC_3 & 0 & 0.189 & 0 & 0 \\ SC_4 & 0 & 0.255 & 0 & 0 \\ SC_9 & 0 & 0.189 & 0 & 0 \end{bmatrix}$$ (4)
The resulting eigenvectors obtained from the pair-wise comparisons matrix, \(W_{33}\), are:

\[
\begin{bmatrix}
SC_1 & SC_2 & SC_3 & SC_4 & SC_5 & SC_6 & SC_7 & SC_8 & SC_9 & SC_{10} & SC_{11} & SC_{12} & SC_{13} & SC_{14} & SC_{15} & SC_{16} & SC_{17} & SC_{18} & SC_{19} \\
0.117 & & & & & & & & & & & & & & & & & & & \\
0.100 & & & & & & & & & & & & & & & & & & & \\
0.152 & & & & & & & & & & & & & & & & & & & \\
0.119 & & & & & & & & & & & & & & & & & & & \\
0.039 & & & & & & & & & & & & & & & & & & & \\
0.043 & & & & & & & & & & & & & & & & & & & \\
0.015 & & & & & & & & & & & & & & & & & & & \\
0.018 & & & & & & & & & & & & & & & & & & & \\
0.037 & & & & & & & & & & & & & & & & & & & \\
0.031 & & & & & & & & & & & & & & & & & & & \\
0.035 & & & & & & & & & & & & & & & & & & & \\
0.022 & & & & & & & & & & & & & & & & & & & \\
0.022 & & & & & & & & & & & & & & & & & & & \\
0.072 & & & & & & & & & & & & & & & & & & & \\
0.026 & & & & & & & & & & & & & & & & & & & \\
0.017 & & & & & & & & & & & & & & & & & & & \\
0.095 & & & & & & & & & & & & & & & & & & & \\
0.006 & & & & & & & & & & & & & & & & & & & \\
0.034 & & & & & & & & & & & & & & & & & & & \\
\end{bmatrix}
\]

Equation 4 indicates that the weights of the sub-criteria via limit super-matrix determination are \(SC_1\) (0.117), \(SC_2\) (0.100), \(SC_3\) (0.152), \(SC_4\) (0.119), \(SC_5\) (0.039), \(SC_6\) (0.043), \(SC_7\) (0.015), \(SC_8\) (0.018), \(SC_9\) (0.037), \(SC_{10}\) (0.031), \(SC_{11}\) (0.035), \(SC_{12}\) (0.022), \(SC_{13}\) (0.022), \(SC_{14}\) (0.072), \(SC_{15}\) (0.026), \(SC_{16}\) (0.017), \(SC_{17}\) (0.095), \(SC_{18}\) (0.006) and \(SC_{19}\) (0.034).

**Step 3: Determining the weight of the WM banks’ performance by using GRA**

The weights are determined for 11 experts with each respondent using Saaty’s relative importance scale and averaging their scale to assess candidates, and then establish a decision making matrix as shown in Table 4.

The nineteen sub-criteria are “the larger, the better” (Appendix B). Accordingly, the referential series can be \(X_9 = (0.436, 0.331, 0.299, 0.299, 0.345, 0.428, 0.426, 0.500, 0.428, 0.347, 0.333, 0.338, 0.343, 0.434, 0.488, 0.331, 0.400, 0.293, and 0.373)\). The WM banks are \(X_1, X_2, X_3\) and \(X_6\).

The candidates are \(X_1, X_2, X_3\) and \(X_6\). Data are normalized for nineteen sub-criteria” from Appendix B, and then we have \(\Delta_{ij}(j)\). Table 5 summarizes the normalization data and Table 6 displays the \(\Delta_{ij}(j)\). The relational coefficients, \(\gamma_{ij}(j)\) of the compared series are shown in Table 7.

**Step 4: Final performance score**

Since the sub-criteria weights have been obtained from...
Table 2. The super-matrix.

| Goal  | C1 | C2 | C3 | C4 | SC1 | SC2 | SC3 | SC4 | SC5 | SC6 | SC7 | SC8 | SC9 | SC10 | SC11 | SC12 | SC13 | SC14 | SC15 | SC16 | SC17 | SC18 | SC19 |
|-------|----|----|----|----|-----|-----|-----|-----|-----|-----|-----|-----|-----|------|------|------|------|------|------|------|------|------|------|------|
| C1    | 0.392 | 0 | 0.500 | 0.407 | 0.458 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| C2    | 0.301 | 0.391 | 0 | 0.370 | 0.286 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| C3    | 0.184 | 0.330 | 0.250 | 0 | 0.256 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| C4    | 0.123 | 0.278 | 0.250 | 0.224 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| SC1   | 0   | 0.251 | 0 | 0 | 0.093 | 0.163 | 0.123 | 0.039 | 0.224 | 0.149 | 0.125 | 0 | 0.286 | 0 | 0.093 | 0.286 | 0 | 0.070 | 0.077 | 0 | 0.222 | 0 |
| SC2   | 0   | 0.164 | 0 | 0 | 0.170 | 0.106 | 0.170 | 0.027 | 0.131 | 0.051 | 0.079 | 0.200 | 0.143 | 0 | 0.047 | 0.143 | 0 | 0.070 | 0.039 | 0 | 0.111 | 0 |
| SC3   | 0   | 0.421 | 0 | 0 | 0.280 | 0.221 | 0.136 | 0.046 | 0.224 | 0.118 | 0.139 | 0 | 0.286 | 0 | 0.122 | 0.286 | 0 | 0.119 | 0.077 | 0 | 0.222 | 0 |
| SC4   | 0   | 0.164 | 0 | 0 | 0.036 | 0.226 | 0.117 | 0.049 | 0.198 | 0.118 | 0.139 | 0.400 | 0.286 | 0 | 0.111 | 0.286 | 0 | 0.023 | 0.077 | 0 | 0.222 | 0 |
| SC5   | 0   | 0   | 0.112 | 0 | 0.066 | 0 | 0 | 0 | 0 | 0.057 | 0.100 | 0 | 0 | 0 | 0.024 | 0 | 0 | 0.033 | 0.039 | 0 | 0 | 0.667 |
| SC6   | 0   | 0   | 0.255 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.207 | 0 | 0 | 0 | 0.158 |
| SC7   | 0   | 0   | 0.189 | 0 | 0 | 0 | 0 | 0 | 0 | 0.210 | 0 | 0 | 0 | 0 | 0 | 0.084 | 0 | 0 | 0 | 0 | 0 | 0 | 0.222 |
| SC8   | 0   | 0   | 0.256 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.198 | 0.299 | 0 | 0 | 0 | 0 | 0 | 0 |
| SC9   | 0   | 0   | 0.189 | 0 | 0 | 0 | 0 | 0 | 0 | 0.140 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.222 |
| SC10  | 0   | 0   | 0 | 0.147 | 0 | 0 | 0 | 0 | 0 | 0 | 0.189 | 0 | 0 | 0 | 0 | 0 | 0.066 | 0.0649 | 0 | 0 | 0 | 0 | 0 |
| SC11  | 0   | 0   | 0.255 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.189 | 0 | 0 | 0 | 0 | 0 | 0 | 0.121 | 0 | 0 | 0 | 0 | 0.154 |
| SC12  | 0   | 0   | 0 | 0.197 | 0 | 0.065 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| SC13  | 0   | 0   | 0 | 0.147 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.045 | 0 | 0 | 0 | 0 | 0.071 |
| SC14  | 0   | 0   | 0 | 0.255 | 0 | 0.092 | 0.285 | 0.064 | 0.043 | 0.224 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.015 | 0 | 0 | 0 |
| SC15  | 0   | 0   | 0 | 0 | 0.320 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.561 | 0 | 0 | 0 |
| SC16  | 0   | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.105 |
| SC17  | 0 | 0 | 0 | 0 | 0.095 | 0.057 | 0 | 0.391 | 0.189 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.054 | 0 | 0 | 0 | 0 | 0.081 |
| SC18  | 0 | 0 | 0 | 0 | 0.243 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.075 | 0 | 0 | 0 | 0 | 0 |
| SC19  | 0 | 0 | 0 | 0 | 0.142 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.022 | 0 | 0 | 0 | 0 | 0.667 |

ANP, the GRG weights can be derived. Table 8 summarizes those results. The performance with respect to performance can be summarized as follows: Bank D (0.601) > Bank C (0.553) > Bank A (0.535) > Bank B (0.522). Therefore, Bank D performs the best.

CONCLUSIONS

The main benefit of the development of the performance measurement system is to provide a structure for performance measurement in WM banks. The model developed here, structures the performance measurement problem in a hierarchical form and links competitive strategies, critical areas and performance measures. In addition, the model combines two different approaches developed in the literature. By adopting a similar approach with integrated performance improvident models, the areas of success are defined to focus on current activities and decisions instead of operational results, which are outcomes of previous activities and decisions. On the other hand, an overall performance score is obtained with the application of a multi-criteria approach. In obtaining the overall performance score, the multi-criteria approach combines the measured performance values for separate measures.

The BSC and performance measures for WM banks with different strategies are likely to be different. As a strategic management system, a
Table 3. The limit super-matrix.

<table>
<thead>
<tr>
<th>Goal</th>
<th>Criteria</th>
<th>Sub-criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>C_1</td>
<td>0.003</td>
<td>0.003 0.003 0.003 0.003</td>
</tr>
<tr>
<td>C_2</td>
<td>0.003</td>
<td>0.003 0.003 0.003 0.003</td>
</tr>
<tr>
<td>C_3</td>
<td>0.002</td>
<td>0.002 0.002 0.002 0.002</td>
</tr>
<tr>
<td>C_4</td>
<td>0.002</td>
<td>0.002 0.002 0.002 0.002</td>
</tr>
</tbody>
</table>

Table 4. The decision making matrix.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
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<th></th>
<th></th>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.159</td>
<td>0.241</td>
<td>0.209</td>
<td>0.239</td>
<td>0.209</td>
<td>0.164</td>
<td>0.231</td>
<td>0.167</td>
<td>0.164</td>
<td>0.204</td>
<td>0.167</td>
<td>0.205</td>
<td>0.243</td>
<td>0.195</td>
<td>0.488</td>
<td>0.331</td>
<td>0.400</td>
<td>0.293</td>
<td>0.373</td>
</tr>
<tr>
<td>B</td>
<td>0.159</td>
<td>0.241</td>
<td>0.239</td>
<td>0.209</td>
<td>0.198</td>
<td>0.175</td>
<td>0.148</td>
<td>0.167</td>
<td>0.175</td>
<td>0.347</td>
<td>0.333</td>
<td>0.338</td>
<td>0.343</td>
<td>0.434</td>
<td>0.153</td>
<td>0.188</td>
<td>0.178</td>
<td>0.207</td>
<td>0.146</td>
</tr>
<tr>
<td>C</td>
<td>0.247</td>
<td>0.188</td>
<td>0.253</td>
<td>0.253</td>
<td>0.345</td>
<td>0.428</td>
<td>0.426</td>
<td>0.500</td>
<td>0.428</td>
<td>0.246</td>
<td>0.167</td>
<td>0.169</td>
<td>0.172</td>
<td>0.177</td>
<td>0.201</td>
<td>0.241</td>
<td>0.144</td>
<td>0.207</td>
<td>0.277</td>
</tr>
<tr>
<td>D</td>
<td>0.436</td>
<td>0.331</td>
<td>0.299</td>
<td>0.299</td>
<td>0.248</td>
<td>0.233</td>
<td>0.195</td>
<td>0.167</td>
<td>0.233</td>
<td>0.204</td>
<td>0.333</td>
<td>0.288</td>
<td>0.243</td>
<td>0.195</td>
<td>0.158</td>
<td>0.241</td>
<td>0.278</td>
<td>0.293</td>
<td>0.205</td>
</tr>
</tbody>
</table>

WM bank's BSC should include action plans for achieving strategic goals and targets for performance measures. In the case where WM banks have different BSCs and performance measures, one can still apply ANP to prioritize performance perspectives and measures for all...
Table 5. Summary of normalization data.

<table>
<thead>
<tr>
<th>Bank selection</th>
<th>Sub-criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>X(j) j = 1, 2, ..., 19</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Reference series (i=0)</th>
<th>SC1</th>
<th>SC2</th>
<th>SC3</th>
<th>SC4</th>
<th>SC5</th>
<th>SC6</th>
<th>SC7</th>
<th>SC8</th>
<th>SC9</th>
<th>SC10</th>
<th>SC11</th>
<th>SC12</th>
<th>SC13</th>
<th>SC14</th>
<th>SC15</th>
<th>SC16</th>
<th>SC17</th>
<th>SC18</th>
<th>SC19</th>
</tr>
</thead>
<tbody>
<tr>
<td>A (i=1)</td>
<td>1.000</td>
<td>0.629</td>
<td>1.000</td>
<td>0.667</td>
<td>0.925</td>
<td>1.000</td>
<td>0.701</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>0.787</td>
<td>0.585</td>
<td>0.930</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>B (i=2)</td>
<td>1.000</td>
<td>0.629</td>
<td>0.667</td>
<td>1.000</td>
<td>1.000</td>
<td>0.958</td>
<td>1.000</td>
<td>0.958</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>1.000</td>
<td>1.000</td>
<td>0.867</td>
<td>1.000</td>
<td>1.000</td>
</tr>
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<td>C (i=3)</td>
<td>0.682</td>
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<td>0.511</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.706</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>0.857</td>
<td>0.629</td>
<td>1.000</td>
<td>1.000</td>
<td>0.423</td>
<td></td>
</tr>
<tr>
<td>D (i=4)</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.659</td>
<td>0.739</td>
<td>0.831</td>
<td>1.000</td>
<td>0.739</td>
<td>1.000</td>
<td>0.296</td>
<td>0.585</td>
<td>0.930</td>
<td>0.985</td>
<td>0.629</td>
<td>0.477</td>
<td>0.000</td>
<td>0.740</td>
<td></td>
</tr>
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</table>

Table 6. Resultants of $\Delta_{ij}(j)$.

<table>
<thead>
<tr>
<th>Bank selection</th>
<th>Sub-criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>X(j) j = 1, 2, ..., 19</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SC1</th>
<th>SC2</th>
<th>SC3</th>
<th>SC4</th>
<th>SC5</th>
<th>SC6</th>
<th>SC7</th>
<th>SC8</th>
<th>SC9</th>
<th>SC10</th>
<th>SC11</th>
<th>SC12</th>
<th>SC13</th>
<th>SC14</th>
<th>SC15</th>
<th>SC16</th>
<th>SC17</th>
<th>SC18</th>
<th>SC19</th>
</tr>
</thead>
<tbody>
<tr>
<td>A (i=1)</td>
<td>0.976</td>
<td>0.368</td>
<td>1.000</td>
<td>0.139</td>
<td>0.776</td>
<td>0.958</td>
<td>0.703</td>
<td>0.915</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>0.776</td>
<td>0.776</td>
<td>0.882</td>
<td>0.000</td>
<td>0.247</td>
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</tr>
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<td>0.474</td>
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<td>0.831</td>
<td>1.000</td>
<td>1.000</td>
<td>0.964</td>
<td>0.969</td>
<td>0.134</td>
<td>0.134</td>
<td>0.831</td>
<td>0.831</td>
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<td>0.000</td>
<td>1.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.727</td>
<td>0.727</td>
<td>0.000</td>
<td>0.000</td>
<td>1.000</td>
<td>0.859</td>
<td>0.784</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>D (i=4)</td>
<td>0.000</td>
<td>0.000</td>
<td>0.526</td>
<td>0.229</td>
<td>1.000</td>
<td>0.623</td>
<td>0.941</td>
<td>1.000</td>
<td>0.789</td>
<td>0.000</td>
<td>0.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>0.638</td>
<td>0.503</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Table 7. Summary of the resultant of the relational coefficients $\gamma_{ij}(j)$.

<table>
<thead>
<tr>
<th>Bank selection</th>
<th>Sub-criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>X(j) j = 1, 2, ..., 19</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SC1</th>
<th>SC2</th>
<th>SC3</th>
<th>SC4</th>
<th>SC5</th>
<th>SC6</th>
<th>SC7</th>
<th>SC8</th>
<th>SC9</th>
<th>SC10</th>
<th>SC11</th>
<th>SC12</th>
<th>SC13</th>
<th>SC14</th>
<th>SC15</th>
<th>SC16</th>
<th>SC17</th>
<th>SC18</th>
<th>SC19</th>
</tr>
</thead>
<tbody>
<tr>
<td>A (i=1)</td>
<td>0.333</td>
<td>0.443</td>
<td>0.333</td>
<td>0.429</td>
<td>0.351</td>
<td>0.333</td>
<td>0.416</td>
<td>0.333</td>
<td>0.333</td>
<td>0.333</td>
<td>0.389</td>
<td>0.461</td>
<td>0.350</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>B (i=2)</td>
<td>0.333</td>
<td>0.443</td>
<td>0.429</td>
<td>0.333</td>
<td>0.333</td>
<td>0.343</td>
<td>0.333</td>
<td>0.333</td>
<td>0.343</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>0.333</td>
<td>0.333</td>
<td>0.366</td>
</tr>
<tr>
<td>C (i=3)</td>
<td>0.423</td>
<td>0.333</td>
<td>0.495</td>
<td>0.495</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>0.415</td>
<td>0.333</td>
<td>0.333</td>
<td>0.333</td>
<td>0.333</td>
<td>0.333</td>
<td>0.333</td>
<td>0.369</td>
<td>0.443</td>
<td>0.333</td>
</tr>
<tr>
<td>D (i=4)</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>0.431</td>
<td>0.404</td>
<td>0.376</td>
<td>0.333</td>
<td>0.404</td>
<td>0.333</td>
<td>0.333</td>
<td>0.333</td>
<td>0.333</td>
<td>0.333</td>
<td>0.333</td>
<td>0.333</td>
<td>0.333</td>
<td>0.333</td>
</tr>
</tbody>
</table>

interdependencies among WM banks. Each WM bank's performance should then be evaluated in terms of its goals (targets for performance measures), which are in alignment with its strategies. The performance priority of each WM bank against its scorecard or performance
targets, as determined by ANP, can then be used to establish the relative performance priorities of WM banks, which have different strategies, BSCs, and performance measures.

The application of the ANP and GRA approach provides Taiwan’s banks with a more accurate and realistic performance score. In a possible application of the proposed performance measurement model, a WM bank can see its overall performance, detect its weak areas in which its performance scores are lower than the other WM banks’ average, and develop necessary programs to close the performance gaps in weak areas. The model provides not only the performance scores, but also weights for the areas of success. Lower weighted areas are considered to be more important for the WM banks and are selected for improvement. Finally, we recommend that bank managers or financial sector analysts apply this model to evaluate the performance of the WM banks.

Further research focusing on the behavioral impact of ANP decisions on bank managers would provide better insights into the value of ANP to decision makers, and the research framework could include not only the risk measurement and management of the bank but also its corporate governance structure in order to improve the WM markets. Nevertheless, ANP is a method that has seen a number of applications, and has especially served as an aid to decision-making where multiple objectives and multiple viewpoints, such as those related to a BSC, prevail.

REFERENCES


Table 8. Summary of the GRG $\Gamma_{bi}$.

<table>
<thead>
<tr>
<th>Bank selection</th>
<th>$\Gamma_{bi}$</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.535</td>
<td>3</td>
</tr>
<tr>
<td>B</td>
<td>0.522</td>
<td>4</td>
</tr>
<tr>
<td>C</td>
<td>0.553</td>
<td>2</td>
</tr>
<tr>
<td>D</td>
<td>0.601</td>
<td>1</td>
</tr>
</tbody>
</table>


APPENDIX

Appendix A

Let the components of a decision system be $C_k, k = 1, \ldots, n$, and each component $k$ have $m_k$ elements, denoted by $e_{k1}, e_k, \ldots, e_{km_k}$. A standard form of a super-matrix can be expressed as in the following formula (Saaty, 1996):

$$W = \begin{bmatrix}
C_1 & \cdots & C_i & \cdots & C_{i*} \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
\vdots & & \ddots & \ddots & \vdots \\
C_{i*} & & \cdots & \ddots & \vdots \\
C_{*} & & \cdots & \cdots & \ddots \\
\end{bmatrix},$$

$$e_{i1} \begin{bmatrix}
W_{1i} & \cdots & W_{ik} & \cdots & W_{i*} \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
\vdots & & \ddots & \ddots & \vdots \\
W_{i*} & & \cdots & \ddots & \vdots \\
W_{*} & & \cdots & \cdots & \ddots \\
\end{bmatrix}.$$

As an example, the super-matrix representation of a hierarchy with three levels is as follows (Saaty, 1996):

$$W_h = \begin{bmatrix}
0 & 0 & 0 \\
W_{21} & 0 & 0 \\
0 & W_{32} & 0 \\
\end{bmatrix},$$

where $W_{21}$ is a vector that represents the impact of the goal on the criteria, and $W_{32}$ is a matrix that represents the impact of sub-criteria on each of the criteria, with the entries of zeros corresponding to those elements that have no influence. For the above example, if the criteria were interrelated among themselves, a network would replace the hierarchy. The $W_{22}$ and $W_{33}$ would indicate the interdependency, and the super-matrix would be:

$$W_n = \begin{bmatrix}
0 & 0 & 0 \\
w_{21} & w_{22} & 0 \\
0 & w_{32} & w_{33} \\
\end{bmatrix}.$$

Appendix B

1. Upper-bound effectiveness measuring (that is, the larger, the better):

$$x^*(j) = \frac{x_i(j) - \min_j x_i(j)}{\max_j x_i(j) - \min_j x_i(j)}$$

2. Lower-bound effectiveness measuring (that is, the smaller, the better):
3. Moderate effectiveness measuring (that is, the nominal, the best):

\[
x_i^* (j) = \frac{\max x_i(j) - x_i(j)}{\max x_i(j) - \min x_i(j)}
\]

If \( \min x_i(j) \leq x_{ob}(j) \leq \max x_i(j) \),

then \( x_i^* (j) = \frac{|x_i(j) - x_{ob}(j)|}{\max x_i(j) - \min x_i(j)} \),

If \( \max x_i(j) \leq x_{ob}(j) \), then \( x_i^* (j) = \frac{x_i(j) - \min x_i(j)}{x_{ob}(j) - \min x_i(j)} \),

If \( x_{ob}(j) \leq \min x_i(j) \), then \( x_i^* (j) = \frac{\max x_i(j) - x_i(j)}{\max x_i(j) - x_{ob}(j)} \),

where \( x_{ob}(j) \) is the objective value of entity \( j \).